Pergamon

Int. J. Solids Structures Vol. 32, No. 16, pp. 2369–2376, 1995

0020–7683/95 $9.50 + .00

0020–7683(94)00226–6

AXI-SYMMETRICAL TRANSIENT CONTACT PROBLEM FOR SLIDING BODIES WITH HEAT GENERATION

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(*Received* 3 *December* 1993; *in revised form* 29 *August* 1994)

Abstract—A transient contact problem with frictional heating for two sliding half-spaces is considered. One of the two half-spaces is assumed to be slightly curved to give a Hertzian initial pressure distribution; the other is a rigid non-conductor. The problem is formulated in terms of one integral equation with unknown heat flux at the interface. The equation is solved numerically, using appropriately chosen Green's functions. Contact pressure and temperatures are found for various values of an independent parameter—the ratio of initial width of contact to the width in the steady state.

INTRODUCTION

As a result of heat generation due to the action of friction forces, contact conditions deteriorate: the nominal contact area decreases, the temperature rises, the irregularity of its distribution (over the friction surface) increases, and so on. These processes progress quickly and after a short while cause the failure of the whole system. Extreme temperature estimates over actual contact spots can be calculated and measured. It is rather difficult to find the maximum temperature at the frictional contact spots since their size is extremely small, but the inertia of the most sensitive thermal couples exceeds the time of existence of contact spots by more than one order of magnitude [see, e.g. Chichinadze *et al.* (1979)]. In these conditions the most convenient and least difficult method for estimating temperature and pressure in the frictional contact is by calculation. Here the calculation model must take into account both the conditions in the frictional contact and the discrete character of the interaction. For example, during intensive momentary breakage the convective heat transfer does not exceed 2–3% of the heat quantity generated in the contact [see, e.g. Chichinadze *et al.* (1986)], and in this case we may neglect the heat exchange over the free surfaces of the system and significantly simplify the problem.

Insofar as temperature change and pressure concentration are unsteady processes, the solution of contact problems of transient thermoelasticity with friction heat generation must be considered. Plane problems have been considered in detail in Azarkhin and Barber (1985, 1987). The axisymmetrical contact problem of uniform sliding of an elastic sphere over the surface of an undeforming insulated half-space was discussed for the first time in the work of Barber (1980). However, the solution was obtained under the assumption that the contact pressure distribution could be described by Hertz formulas during all the processes of interaction. It was shown that this gave an error of about 22% in the determination of the radius of the contact area. In the present article the same problem has been solved without the above-mentioned limitations.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

We consider an axisymmetrical transient contact problem for two semi-limited bodies, one of which is sliding over the surface of the other with steady velocity v and pressing into it with a force **P** (Fig. 1.). Sliding is accompanied by heat generation over the contact interface in the form of a heat flux

$$q(r,t) = fvp(r,t), \quad r \leqslant a(t), \quad t > 0 \tag{1}$$

(where a is the radius of contact area, p is the contact pressure, f is the coefficient of friction) directed into the moving body. It is considered that an immovable body is a rigid thermoinsulator; the convective heat transfer from the free surface of contacting bodies is absent. There are no coupling tangential and normal tractions. This assumption does not mean that the tangential traction on the surface is neglected. Indeed, the work done against these tractions is the source of the heat generation. However, the elastic displacements normal to the surface, caused by the tangential tractions, are much smaller than those produced by the normal tractions, and the coupling effect is negligible.

In such a statement the solution to the problem can be presented as the superposition of two problems:

(1) the finding of temperature stresses and deformations in the heat-conducting body due to its heating by the heat flux (1);

(2) the solution to the isothermal problem of the contact of a body with the thermally deformed surface and the rigid non-heat conducting half-space.

Thus, we shall write the normal displacements of points at the boundary of an elastic half-space in the form

$$u_z(r,t) = u_z^e(r,t) + u_z^t(r,t), \quad r > 0, \quad t > 0. \tag{2}$$

The problem of the elasticity for bodies in contact will be considered in a quasi-static statement. In this case for u_z^e we have the expression [see, e.g. Johnson (1987)]

$$u_z^e(r,t) = \frac{(1-\nu)}{\pi\mu}\int_0^{a(t)} p(s,t)L(r,s)\,\mathrm{d}s, \quad r > 0, \quad t > 0 \tag{3}$$

$$L(r,s) = \frac{2s}{r+s}K\left[\frac{4rs}{(r+s)^2}\right],$$

(where $K(\cdot)$ is the complete elliptic integral of the second kind, μ is the shear modulus, and ν is the Poisson's ratio).

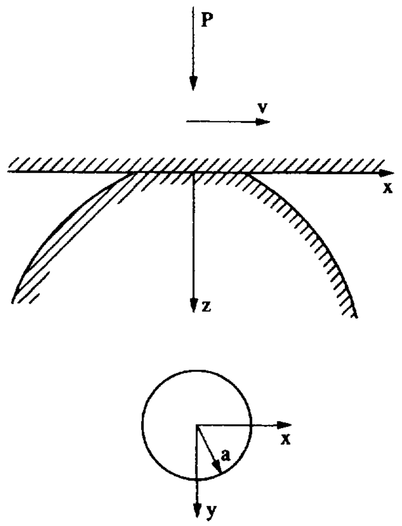

Fig. 1. Model of thermal contact.

The solution of the transient heat equation for heat sources distributed with density $q(r, t)$ in the circle $r \leqslant a(t)$ over the half-space surface has the form [see, e.g. Carslaw and Jeager (1959)]

$$T(r, t) = \frac{1}{4\rho c(\pi k)^{3/2}} \int_0^t \int_0^{a(t)} \int_0^{2\pi} q(s, \tau) \exp\left[-\frac{r^2 - 2rs\cos(\theta) + s^2}{4k(t-\tau)} \right] \frac{s\,\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}\tau}{(t-\tau)^{3/2}}, \tag{4}$$

where T is the temperature, k is the thermal diffusivity, p is the density, and c is the specific heat. Normal displacements correspond to the temperature field (4) [see, e.g. Barber (1972)]

$$u_z^t(r, t) = -\frac{\delta}{4\pi} \int_0^t \int_0^{a(t)} \int_0^{2\pi} q(s, \tau)\Phi\left[1.5; 2; -\frac{r^2 - 2rs\cos(\theta) + s^2}{4k(t-\tau)} \right] \frac{s\,\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}\tau}{(t-\tau)}, \tag{5}$$

(where $\delta = \alpha(1+\nu)/\lambda$ is the coefficient of thermal distortivity, λ is the thermal conductivity, α is the coefficient of linear thermal expansion, and Φ is the degenerated hypergeometric function).

Considering that the surface of the conducting elastic body is slightly distorted with radius curvature, the condition of the contact of bodies can be written

$$g(r, t) \equiv u_z(r, t) - \Delta(t) + r^2/2R = 0, \quad r \leqslant a(t), \quad t > 0 \tag{6}$$

where $\Delta(t)$ is the approach of bodies as two rigid solids.

Substituting expressions (3) and (5) into correlations (2) and (6), we obtain the integral equation of the problem:

$$\frac{(1-\nu)}{\pi\mu} \int_0^{a(t)} p(s, t) L(r, s)\,\mathrm{d}s - \frac{\delta}{4\pi} \int_0^t \int_0^{a(t)} \int_0^{2\pi} q(s, \tau)\Phi$$

$$\times\left[1.5; 2; -\frac{r^2 - 2rs\cos(\theta) + s^2}{4k(t-\tau)} \right] \frac{s\,\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}\tau}{(t-\tau)} = \Delta(t) - r^2/2R, \quad r \leqslant a(t), \quad t > 0. \tag{7}$$

To the integral equation (7) it is necessary to add the condition of equilibrium of bodies

$$2\pi \int_0^{a(t)} p(s, t) s\,\mathrm{d}s = \mathbf{P} \tag{8}$$

and physical inequalities

$$p(r, t) \geqslant 0, \quad r \leqslant a(t), \quad t > 0 \tag{9}$$

$$g(r, t) > 0, \quad r > a(t), \quad t > 0. \tag{10}$$

In view of connection (1), the system of integral equations (7) and (8) allows definition of the contact pressure $p(r, t)$, the heat flux $q(r, t)$, and the approach of bodies $\Delta(t)$. The inequalities (9) and (10) allow the contact radius $a(t)$ to be found.

Denote

$$r = a_*\tilde{r}, \quad s = a_*\tilde{s}, \quad t = (a_*^2/k)\tilde{t}, \quad \tau = (a_*^2/k)\tilde{\tau}, \quad \tilde{a}(t) = a(t)/a_*,$$

$$p = (\mathbf{P}/a_*^2)\tilde{p}, \quad T = (fv\mathbf{P}/\lambda a_*)\tilde{T}, \quad q = (fv\mathbf{P}/a_*^2)\tilde{q}. \tag{11}$$

The value

$$a_* = \frac{\pi\lambda(1-\nu)}{1.566\alpha\mu f v(1+\nu)}; \tag{12}$$

this is a critical value of the contact radius as the temperature field (4) reaches steady state [see, e.g. Barber (1976)]. As we see from eqn (12), a_* does not depend upon the compressible force **P** and is a boundary value of the contact radius at unlimited increase in **P**. Note that in isothermal cases such a limit does not exist.

Taking into account notations (11), the system of integral equations (7) and (8) will become (omitting waves over the values)

$$\frac{1}{\pi}\int_0^{a(t)} q(s,t)L(r,s)\,\mathrm{d}s - \frac{1}{6.264}\int_0^t\int_0^{a(t)}\int_0^{2\pi} q(s,\tau)\Phi\left[1.5,2,-\frac{r^2-2rs\cos(\theta)+s^2}{4(t-\tau)}\right]\frac{s\,\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}\tau}{(t-\tau)}$$
$$= 0.375a_c^{-1}(\Delta_0(t)-r^2/2a_c^2), \quad r \leqslant a(t), \quad t>0, \tag{13}$$

$$2\pi\int_0^{a(t)} p(s,t)s\,\mathrm{d}s = 1. \tag{14}$$

Here $a_c = a(0)/a_*$ and $\Delta_0(t) = \Delta(t)/\Delta(0)$. The values $a(0)$ and $\Delta(0)$, which correspond to the solution of the isothermal Hertz problem, are given by Johnson (1987):

$$a(0) = \left[\frac{3}{8}\frac{\mathbf{P}R(1-\nu)}{\mu}\right]^{1/3}, \quad \Delta(0) = a^2(0)/R.$$

Thus, the solution of the integral equation systems (13) and (14) depends upon one independent parameter a_c which characterizes the measure of surface coordination of contacting bodies.

Remembering the notation (11) for a dimensionless temperature $T(r,t)$, from eqn (4) we find

$$T(r,t) = \frac{1}{4\pi^{3/2}}\int_0^t\int_0^{a(t)}\int_0^{2\pi} q(s,\tau)\exp\left[-\frac{r^2-2rs\cos(\theta)+s^2}{4k(t-\tau)}\right]\frac{s\,\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}\tau}{(t-\tau)^{3/2}} \tag{15}$$

and eqn (1) will take the form

$$q(r,t) = p(r,t). \tag{16}$$

PREVIEW OF THE ALGORITHM

We now build the number solution of the integral equations (13) and (14) by the method of piecewise-constant approximation [see, e.g. Marchuk and Agoshkov (1981)]. For this purpose we shall divide the temporal interval $[0,t]$ into l parts of the length $\delta t = t/l$ by points $0 = t_0 < t_1 < \cdots < t_l = t$. We shall divide the contact strip into n concentric annuli of radii a, where $0 = a_0 < a_1 < \cdots < a_{n-1} < a_n = a(t)$. Let us assume that the heat flux is constant and equal to q_{ij} in every temporal-spatial region $[t_{j-1},t_j]\times[a_{i-1},a_i]$. Then at the moment of time $t = t_l$ we obtain a discretized analogue of integral equations (14) and (15):

$$\sum_{i=1}^{n} q_{il}\left[\pi^{-1}b_{ik} - \frac{8\pi}{6.264}c_{ilkl}\right] = 0.375a_c[\Delta_0(t_l) - r_k^2/2a_c^2]$$

$$+\frac{8\pi}{6.264}\sum_{i=1}^{n}\sum_{j=1}^{l-1} q_{ij}c_{ijkl}, \quad (k = 1, 2, \ldots, n), \qquad (17)$$

$$2\pi \sum_{i=1}^{n} q_{il} r_i \delta a = 1, \qquad (18)$$

where

$$b_{ik} = F_0(r_k, a_i) - F_0(r_k, a_{i-1})$$

$$c_{ijkl} = \begin{cases} t_{jl}^{-}[(A_{ijl}^{-})^2 F_1(R_{jkl}^{-}, A_{ijl}^{-}) - (A_{i-1,jl}^{-})^2 F_1(R_{jkl}^{-}, A_{i-1,jl}^{-})] \\ -t_{jl}^{+}[(A_{ijl}^{+})^2 F_1(R_{jkl}^{+}, A_{ijl}^{+}) - (A_{i-1,jl}^{+})^2 F_1(R_{jkl}^{+}, A_{i-1,jl}^{+})], \quad j \neq l, \\ -t_{jl}^{+}[(A_{ijl}^{+})^2 F_1(R_{jkl}^{+}, A_{ijl}^{+}) - (A_{i-1,jl}^{+})^2 F_1(R_{jkl}^{+}, A_{i-1,jl}^{+})], \quad j = l, \end{cases}$$

$$R_{jkl}^{\pm} = 0.5r_k/(t_{jl}^{\pm})^{1/2}, \quad r_k = a_k - 0.5\delta a, \quad k = 1, 2, \ldots, n,$$

$$A_{ijl}^{\pm} = 0.5a_i/(t_{jl}^{\pm})^{1/2}, \quad a_i = i\delta a, \quad i = 1, 2, \ldots, n; \quad \delta a = a/n,$$

$$t_{jl}^{\mp} = (l - j \pm 0.5)\delta t, \quad j = 1, 2, \ldots, l.$$

The influence function $F_0(r, a)$, on the basis of Johnson (1987), has the form

$$F_0(r, a) = \begin{cases} 2aE(r/a), & r \leqslant a \\ 2r[E(a/r) - (1 - a^2/r^2)K(a/r)], & r > a \end{cases}$$

where $K(\cdot)$, $E(\cdot)$ are complete elliptic integrals of the first and second kind.

The function $F_1(R, A)$ has been found in Barber (1972) as:

$$F_1(R, A) = \begin{cases} \ln(A/2) + 0.5\left[\gamma + \frac{R^2}{A^2}\right] + \sum_{i=1}^{\infty} \frac{(2i+1)!!(-A^2)^i}{(2i+2)!!i!i} \sum_{j=0}^{i} (C_j^i)^2 \frac{(R/A)^{2j}}{(i-j+1)}, & R < A, \\ \ln(R/2) + 0.5[1+\gamma] + \sum_{i=1}^{\infty} \frac{(2i+1)!!(-R^2)^i}{(2i+2)!!i!i} \sum_{j=0}^{i} (C_j^i)^2 \frac{(A/R)^{2j}}{(j+1)}, & R \geqslant A, \end{cases} \qquad (19)$$

where $\gamma = 0.577216$ is Euler's constant, $C_j^i = i!/(i-j)!/j!$.

The system $n+1$ of linear algebraic equations (17) and (18) allows determination of the unknown values q_{il}, $(i = 1, 2, \ldots, n)$ and $\Delta_0(t_l)$, after which the contact pressure can be found with formula (16).

We tested the present numerical method with problems which have analytical solutions. For example, the algorithm for solving the contact problem was validated using a closed-form solution of the isothermal Hertz contact problem from Johnson (1987). The numerical algorithm gave very good accuracy.

The frictional temperature algorithm was tested by a variety of analytical solutions from Carslaw and Jaeger (1959).

In the choice of the time step, it is necessary to be very careful since too great a time step can lead to inaccuracy and instability. Difficulties can also be encountered if the time step is too small. In this case the solution can lead to convergence on a spurious steady state.

Some guidance as to the appropriate time step δt can be obtained from the parameter $\varepsilon = n\delta a/(2\delta t)^{1/2}$ where δa is a representative measure of the spatial discretization. In the present algorithm ε is required to be less than 2.5 for the convergence of series in eqn (19). At the first time step $n\delta a = a_c$. Therefore $\delta t \geqslant 0.5$ for $a_c = 2.5$. Unfortunately, we could not obtain an asymptotic expression for the functions $F_1(R, A)$ at $A > 2.5$. We hope to do this at a later date.

The contact radius $a(t_l)$ is not initially known, and it is defined by inequalities (9) and (10). If $a(t_l)$ is defined inexactly, then at some points in the contact region a negative contact pressure appears, and interpenetration of materials arises. We liberate points of the first kind from the contact, and introduce points of the second kind into it, and repeat the procedure. The convergence of such an algorithm has been proved in Azarkhin and Barber (1987). Calculations show that no less than five iterations are necessary to receive a relative exactness in 1% in determining $a(t_l)$. We find the temperature in the contact area from the equations

$$T(r_k, t_l) = \sum_{i=1}^{n} \sum_{j=1}^{l} q_{ij} d_{ijkl}.$$

Here

$$d_{ijkl} = \begin{cases} -2t_{jl}^{-}\,[(A_{ijl}^{-})^2 F_2(R_{jkl}^{-}, A_{ijl}^{-}) - (A_{i-1,jl}^{-})^2 F_2(R_{jkl}^{-}, A_{i-1,jl}^{-})] \\ +2t_{jl}^{+}\,[(A_{ijl}^{+})^2 F_2(R_{jkl}^{+}, A_{ijl}^{+}) - (A_{i-1,jl}^{+})^2 F_2(R_{jkl}^{+}, A_{i-1,jl}^{+})], \quad j \neq l, \\ 2t_{jl}^{+}\,[(A_{ijl}^{+})^2 F_1(R_{jkl}^{+}, A_{ijl}^{+}) - (A_{i-1,jl}^{+})^2 F_1(R_{jkl}^{+}, A_{i-1,jl}^{+})], \quad j = l, \end{cases}$$

$$F_2(R, A) = \begin{cases} 2E(R/A)/(\pi A) - \pi^{1/2} \sum_{i=0}^{\infty} \dfrac{(-A^2)^i}{(2i+1)i!} \sum_{j=0}^{i} (C_j^i)^2 \dfrac{(R/A)^{2j}}{(i-j+1)} & R < A, \\ 2R^2 A^{-2}[E(A/R) - (1 - A^2 R^{-2})K(A/R)]/(\pi R) \\ \quad -\pi^{1/2} \sum_{i=0}^{\infty} \dfrac{(-R^2)^i}{(2i+1)i!} \sum_{j=0}^{i} (C_j^i)^2 \dfrac{(A/R)^{2j}}{(j+1)}, & R \geqslant A. \end{cases}$$

RESULTS AND DISCUSSION

The development of the dimensionless contact pressure distribution $\tilde{p}$ in time is plotted in Fig. 2. The curve $\tilde{t} = 0$ conforms to the solution of the corresponding Hertz isothermal problem. We can see that when dimensionless time reaches values larger than 2, the contact

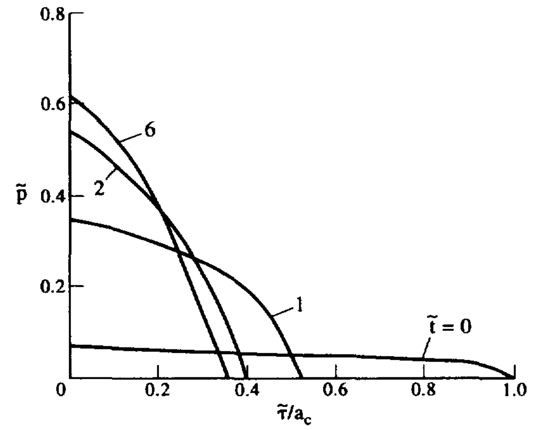

Fig. 2. Normalized pressure distribution $\tilde{p}$, as a function $\tilde{t}$ for $a_c = 2.5$.

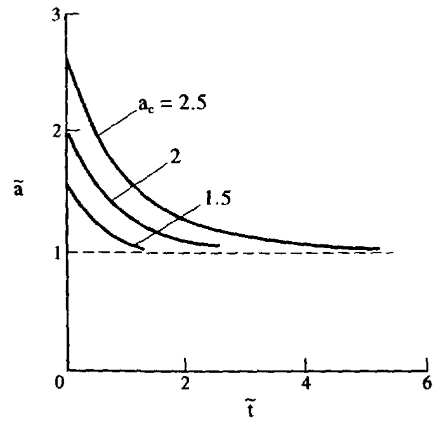

Fig. 3. Reduction of contact area with time.

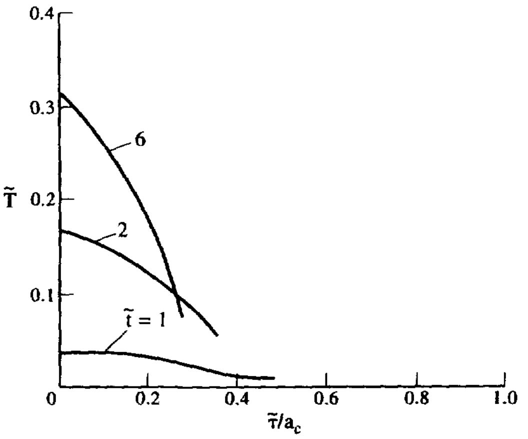

Fig. 4. Development of temperature distribution for $a_c = 2.5$.

pressure does not change significantly. The change of contact area size is plotted in Fig. 3. The value $\tilde{a} = 1$ is noted by a small-dashed line when the temperature field becomes steady. The contact radius at small (<0.5) values $\tilde{t}$ decreases linearly; in this range of $\tilde{t}$ change we may use an approximate solution obtained in Barber (1980):

$$\tilde{a}(\tilde{t}) = \tilde{a}(0) - 0.425\tilde{t}.$$

The distribution of dimensionless contact temperature $\tilde{T}$ for values of the Fourier criterion $\tilde{t} = 1, 2, 6$ is shown in Fig. 4. The results of the present work have shown that heat generation from the action of friction forces in frictional sliding contact leads to a significant (in comparison with the isothermal case) redistribution of contact pressure. As a result of temperature heating, the thermal distortion of the surfaces of elastic bodies takes place, which leads to a decrease in contact area, and contact spots with excessively large pressure and high temperature are formed. In the actual contact areas, the temperature can exceed that permissible for friction couple boundaries, which, in turn, reduces to local adhesion scoring and structure transformations that can propagate all over the friction surface in the process of work.

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